CBSE MATHS 2000 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Let A = {2, 3, 4, 5, 6, 7, 8, 9}. Let R be the relation on A defined by
$\{(x, y) : x \in A, y \in A$ and x divides y}.
Find the domain and range of R.

*2. If a matrix has 8 elements, what are the possible orders it can have?

*3. Evaluate: (i) $\begin{bmatrix}1\\2\\3\end{bmatrix}$ [2 3 4] and (ii) $\begin{bmatrix}1 & -2\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\end{bmatrix}$

*4. If a line makes angle 90^o, 60^o, and 30^o with the positive direction of x, y and z respectively, find itsb direction consines.

*5. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

*6. Check the continuity of the function $f(x) = 2x + 3$ at x = 1.

*7. Evaluate: $\int \cos^4 x dx$.

8. Solve the differential equation:

$$\frac{dy}{dx} = 1 + x + y + xy.$$

9. Find a matrix X such that 2A + B + X = 0, where $A = \begin{bmatrix}-1 & 2\\3 & 4\end{bmatrix}; B = \begin{bmatrix}3 & -2\\1 & 5\end{bmatrix}$.

10. Using differentials, find the approximately value of $\sqrt{26}$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

*11. Let f : {2, 3, 4, 5} → {3, 4, 5, 9} and g: {3, 4, 5, 9} → {7, 11, 15} be functions defined as f = 3, $f(3) = 4$, $f(4) = f(5) = 5$ and $g(3) = g(4)$ and $g(5) = g(9) = 11$. Find go f.

*12. Evaluate: $\int \frac{1+\tan x}{x+\log\sec x}dx$.

Or

Evaluate: $\int \frac{dx}{\sqrt{3-x+x^2}}$.

13. A die is rolled. If the outcome is an even number, what is the probability that it is a prime number?

14. Three bags contain 7 white, 8 red, 9 white, 6 red and 5 white, 7 red balls respectively. One ball, at random, is drawn from each bag. Find the probability that all of them are of the same colour.

15. If $x^p y^q = (x + y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

16. Solve the differential equation:

$(x+2y^2)\frac{dy}{dx} = y$, given that when x = 2, y = 1.

17. Find the projection of $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ on $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

*18. Find the differentiation of $\sqrt{\sin x}$ by first principle.

Or

For the function $f(x) = -2x^2 - 9x^2 - 12x + 1$, find the interval (s):

(a) in which $f(x)$ is increasing.

(b) in which $f(x)$ is decreasing.

*19. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$ in the simplest form where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Or

Prove that: $\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x-x^3}{1`-3x^2}\right), |x| < \frac{1}{\sqrt{3}}$.

20 Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)\ .$$

21. Evaluate: $\int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$.

Or

Evaluate: $\int_{1}^{2}\left(\frac{x-1}{x^2}\right)e^x dx$.

*22. Find the direction cosines of the line passing through the two points (–2, 4, –5) and (1, 2, 3).

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Find λ so that the four points with position vectors $-6\hat{i}+3\hat{j}+2\hat{k}$, $3\hat{i}+\lambda\hat{j}+4\hat{k}$, and $5\hat{i}+7\hat{j}+3\hat{k}$ and $-13\hat{i}+17\hat{j}-\hat{k}$ are coplanar.

24. An unbiased coin is tossed 6 times. Find using Binomial distribution, the probability of getting atleast 5 heads.

Or

A company has two plants to manufacture scooters. Plant-1 manufacture 70% of the scooters and Plant-2 manufactures 30%. AtPlant-1, 80% of the scooters are rated of standard quality and at Plant-2, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it has come from Plant-2.

25. Using matrices, solve the following system of equations for x, y and z:

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

26. A window is in the form of a rectangle surmounted by a semi-circular opening. If the peri meter of the window is 20m, find the dimensions of the window so that the maximum possible light is admitted through the whole opening.

27. Using properties of integrals, evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx$, where $f(x) = \sin|x| + \cos|x|$.

28. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $\vec{r}\cdot(3\hat{i}-5\hat{j}+4\hat{k})+11=0$ and passing through the point (–2, 1, 3).

29. A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital is required to produce one unit of Y. If X and Y are priced at Rs 100 and Rs 120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem, graphically.

ANSWERS

1. Domain of R= {2, 3, 4, 5, 6, 7, 8, 9} = A
   Range of R= {2, 3, 4, 5, 6, 7, 8, 9} = A
2. Possible orders are 1×8, 8×1, 4×2 and 2×4.
3. (i) $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$ (ii) $\begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$
4. $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$
5. $\frac{\pi}{4}$
6. f is continuous at x = 1
7. $\frac{3}{8}x + \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + c$
8. $\log (1 + y) = x + x^2 + c$
9. $\begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$
10. 5.1
11. $(gof)(2) = 7$, $(gof)(3) = 7$, $(gof)(4) = 11$, $(gof)(5) = 11$,
12. $\log|x + \log\sec x| + c$

    or

    $\log\left[\left(x - \frac{1}{2}\right) + \sqrt{3 - x + x^2}\right] + c$
13. $\frac{1}{3}$
14. $\frac{217}{900}$
16. $\frac{x}{y} = 2y + c$
17. $\frac{5}{\sqrt{6}}$
18. $\frac{\cos x}{2\sqrt{\sin x}}$

    Or

    (a) $f(x)$ is increasing in $]-2, -1[$

    (b) $f(x)$ is decreasing in $]-\infty, -2[\cup]-1, \infty[$

19. $\frac{\pi}{4}+\frac{x}{2}$

21. $2\sqrt{x^2+4x+3}-\log\left[x+2+\sqrt{x^2+4x+3}\right]+c$

Or

$\left[e^x\cdot\frac{1}{x}\right]_1^2=\frac{e^2}{2}-e$

22. Required direction cosines are $\frac{3}{\sqrt{77}},\frac{-2}{\sqrt{77}},\frac{8}{\sqrt{77}}$

23. $\lambda = -2$

24. $\frac{7}{64}$

Or

$\frac{27}{83}$

25. x = 1; y = –5; z = –5

26. For maximum possible light, the dimensions of the window are

$2x=\frac{40}{\pi+4}$ and $y=\frac{1}{2}\left[20-(\pi+2)\times\frac{20}{\pi+4}\right]=\frac{20}{\pi+4}$ m

27. $\left[-\left(\cos\frac{\pi}{2}-\cos 0\right)+\left(\sin\frac{\pi}{2}-\sin 0\right)\right]=4$

28. $\lambda=\frac{1}{6}$, Substitute this value of λ in this equation: $[\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})-3]+\lambda[\vec{r}\cdot(3\hat{i}-5\hat{j}+4\hat{k})+11]=0$