Delhi Region

CBSE MATHS 2001 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Let f = {(1, 3), (2, 1), (3, 2)} and g = {(1, 2), (2, 3), (3, 1)}, then find (go$f$) (1).

*2. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$.

*3. Evaluate: $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$

*4. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

*5. Discuss the continuity of the function f defined by

$$f(x) = \begin{cases} x + 2, \text{if } x \le 1 \\ x - 2, \text{if } x > 1 \end{cases} \text{at } x = 1$$

6. Evaluate: $\int \frac{\sec^2(\log x)}{x} dx$.

7. Solve: $(x - 1)\frac{dy}{dx} = 2x^3 y$.

8. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew symmetric matrix, where A^T, where A^T denotes the transpose of A.

9. If $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{k}$, find $|2\vec{b} \times \vec{a}|$.

*10. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. Verify Rolle's Theorem for the function $f(x) = x^2 - x - 12$ in [–3, 4].

Or

Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 30$ is (i) increasing, (ii) decreasing.

12. Evaluate: $\int \sin 5x.\sin 3x\, dx$.

Or

Evaluate: $\int \log(1 + x^2)dx$.

13. Ramesh appears for an interview for two posts A and B for which selection is independent. The probability of his selection for post A is 1/6 and for post B is 1/7. Find the probability that Ramesh is selected for at least one of the posts.

14. Two cards are drawn one by one with out replacement from a well from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

15. Solve: $x\frac{dy}{dx} = y(\log y - \text{lo}x - 1)$.

16. $y = x^{\sin x} + (\sin x)^x$, find $\frac{dy}{dx}$.

17. Find the derivative of $\sin(x^2 + 1)$ w.r.t x from the first principles.

*18. Let T be the set of all triangles in a plane with R a relation in T given by $R = \{T_1\, T_2 : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

19. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

20. Prove, using the properties of determinants.

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

21. Evaluate: $\int \frac{4x+5}{\sqrt{2x^2+x-3}} dx$.

*22. Show that:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\frac{3}{4}.$$

Or

Prove that $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}$

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z + 5 = 0, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$.

Or

If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}, \vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$.

24. Using binomial probability distribution the probability of obtaining "less than 3 heads" when an unbiased coin is tossed 6 times.

Or

A speaks truth in 60% of the cases and B in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact.

25. Using integration, find the area of the triangle ABC where A is (2, 3), B is (4, 7) and C is (6, 2).

26. Solve using matrices: x – y + z = 3; 2x + y – z = 2 and –x – 2y + 2z = 1.

27. show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

28. A manufacturer produces two types steel trunks. He has two machines A and B. The first type of trunk requires 3 hours on machine A and 3 hours on machine B. The second type requires 3 hours on machine A and 2 hours on machine B. machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and second type respectively. How many trunks of each type must he make each day to make maximum profit?

29. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+\hat{j}-2\hat{k}$, verify that

Or

Define the line of shortest distance between two skew lines. Fin the shortest distance and the vector equation of the line of shortest distance between the lines given by

$\vec{r}=(2\hat{j}-3\hat{k})+\lambda(2\hat{i}-\hat{j})$ and

$\vec{r}=(4\hat{i}+3\hat{k})+\mu(3\hat{i}+\hat{j}+\hat{k})$.

ANSWERS

1. 1
2. $X = \frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$
3. 0
4. 11
5. f (x) is not continuous at x = 1
6. tan(log x) + c
9. $6\sqrt{14}$
10. $\frac{7\pi}{6}$

    f will be increasing in]–∞, 1[∪] 2, ∞ [

    f will be decreasing in]1, 2[
12. $\frac{1}{2}\left[\frac{1}{2}\sin 2x - \frac{1}{8}\sin 8x\right] + c$

    Or

    $x(\log)(1+x^2) - 2(x - \tan^{-1} x) + c$
13. $\frac{2}{7}$
14. 1
15. $y = x\, e^{cx}$
16. $x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] + (\sin x)^x[\log \sin x + x \cot x]$
17. $2x \cos (x^2 + 1)$
18. R is an equivalence relation
19. $\frac{\pi^2}{4}$
20. No answer
21. $2\sqrt{2x^2 + x - 3} + 2\sqrt{2}\log\left|x + \frac{1}{4} + \sqrt{x^2 + \frac{1}{2}x - \frac{3}{2}}\right| + c$
23. 7

    Or

    $(\vec{a} - \vec{d}) \parallel (\vec{b} - \vec{c})$
24. $\frac{11}{32}$

Or

They will contradict each other in stating the same fact in 42% cases.

25. 9 sq. units

26. $x=\frac{5}{3}, y=-\frac{4}{3}+k \text{ and } z=k (\forall k \in R)$

28. For getting maximum profit of Rs 165, 3 trucks of each type should be manufactured.

29. Or

Shortest Distance = $PQ=\sqrt{(2-1)^2+(1+1)^2+(-3-2)^2}=\sqrt{30}$

$(2\hat{i}+\hat{j}-3\hat{k})+t(-\hat{i}-2\hat{j}+\bar{k})$