CBSE MATHS 2002 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Let $f : R \to R$ be defined by $f(x) = 3x - 2$ and $g : R \to R$ be defined by $g(x) = \frac{x+2}{3}$. Show $fog = I_R$.

*2. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify $A'A = 1$.

*3. Using the property of determinants, prove that:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

*4. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

5. If $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2, -1, 4]$, verify that $(AB)' = B'A'$.

6. Show that the function $f(x) = 2x - |x|$ is continuous at $x = 0$.

7. If $y = e^x (\sin x + \cos x)$, prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

8. Evaluate: $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$.

9. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$ find $(\vec{a} + 3\vec{b}).(2\vec{a} - \vec{b})$.

10. Evaluate: $\int \frac{x}{(x^2+1)(x+1)} dx$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. If $f(x) = e^x$ and $g(x) = \log x (x > 0)$, show that $fog = gof$.

12. Find the derivative of $\tan x^2$ w.r.t. from the first principle.

13. Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

14. Evaluate: $\int \frac{dx}{\sqrt{7-6x-x^2}}$.

15. Solve the following differential equation:

$$\left(y - x\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right)$$

16. Solve the following differential equation:

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x.$$

17. Four digit numbers are formed by using the digits 1, 2, 3, 4 and 5 without repeating any digit. Find the probability that a number, chosen at random, is an odd number.

18. A bag contains 4 yellow and 5 red balls and another bag contains 6 yellow and 3 red balls. A ball is drawn from the first bag and without seeing its colour, it is put into the second bag. Find the probability that if now a ball is drawn from the second bag, it is yellow in colour.

*19. Write the following functions in the simplest form $\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$

20. Using the properties of determinants, show that:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3.$$

21. Evaluate: $\int_0^{\pi/2} \frac{\sin^2 x\, dx}{\sin x + \cos x}$

Or

Prove that: $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \frac{\pi}{2} - \log 2$

22. The position vectors of two points A and B are $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 2\hat{j} - 4\hat{k}$ respectively. Find the vector equation of the plane passing through B and perpendicular to the vector AB.

Or

For any non-zero vector $\vec{a}$, show that $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Find the shortest distance between the following lines:

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2)\hat{k}$$

and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

24. A factory has three machines X, Y and Z producing 1000, 2000 and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% and Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random and is found defective. What is the probability that this defective bolt has been produced by the machine X?

25. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi+4}$ cm.

26. Using matrix method, solve the following system of equations:

$$x + 2y + z = 7;\ x + 3z = 11;\ 2x - 3y = 1$$

27. A farmer has a supply of chemical fertilizer of type I which contains 10% nitrogen and 6% phosphoric acid and type II fertilizer which contains 5% nitrogen and 10% phosphoric acid. After testing the soil conditions of a field, it is found that at least 14 kg of nitrogen and 14 kg of phosphoric acid is required for a good crop. The fertilizer type I costs Rs 2.00 per kg and the type II costs Rs 3.00 per kg. How many kilograms of each fertilizer be used to meet the requirement and the cost be minimum?

28. Find the value of $\int_0^2 (2x^2 - 3)\ dx$ as limit of sums.

Or

Draw a rough sketch and find the area of the region bounded by the two parabolas $y^2 = 4x$ and $x^2 = 4y$ by using method of integration.

29. Find the angle between the lines whose direction cosines are given by the equations:

$$3l + m + 5n = 0 \text{ and } 6mn - 2nl + 5lm = 0$$

ANSWERS

4. $\frac{\pi}{6}$

8. $\frac{1}{2}e^2 - e$

9. -15

10. $\frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}x - \frac{1}{2}\log|x+1| + c$

12. $\frac{dy}{dx} = 1 \cdot \frac{2x}{\cos x^2 \cos x^2} = 2x\sec^2 x^2$

14. $\int \frac{dx}{\sqrt{4^2-(x+3)^2}} = \sin^{-1}\left(\frac{x+3}{4}\right) + c$

Or

$-\frac{1}{2}\cos(2\tan^{-1}x) + c$

15. $y = k(x+a)(1-ay)$

16. $y = -1 + \tan^{-1}x + ce^{-\tan^{-1}x}$

17. 0.6

18. $\frac{29}{45}$

19. $y = \frac{\pi}{2} - \sec^{-1}x$

20. $x[15x^2 + 10xy - (14x^2 + 10xy)] = x^3$

21. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

22. $2x + 3y + 6z + 28 = 0$

23. The shortest distance between the given lines $= \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \frac{8}{\sqrt{29}}$

24. 0.1

26. x = 2, y = 1, z = 3 is the required solution

27. 100 kg of Type I and 80 kg of Type II fertilizers should be used to meet the desired requirement and minimum cost.

28. $-\frac{2}{3}$

Or

$\frac{16}{3}$ sq. units

29. $\cos^{-1}\left(\frac{1}{6}\right)$