Delhi Region

CBSE MATHS 2003 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Show that the relation R in R defined as R= {(a, b) : a ≤ b} is transitive.

*2. If $A = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB.

*3. Using the property of determinants and without expanding, show that

$$\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$$

4. Differentiate the following w.r.t. x: $\log\left(x + \sqrt{1+x^2}\right)$.

5. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

6. Form the differential equation of the following family of curves:

$$xy = Ae^x + Be^{-x} + x^2$$

7. A balloon which always remains spherical is being inflated by pumping in gas at the rate of 900 cm^3/sec. Find the rate at which the radius of the balloon in increasing when the radius of the balloon is 15 cm.

8. The dot product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k}, \hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

*9. Find the principal values of $\tan^{-1}\left(-\sqrt{3}\right)$.

10. Evaluate: $\int \frac{x\,dx}{(x+2)(3-2x)}$

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. Discuss the continuity of the function:

$$f(x)=\begin{cases} x, & \text{if } \ 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & \text{if } \ x=\frac{1}{2} \\ 1-x & \text{if } \ x>\frac{1}{2} \end{cases} \quad \text{at } x=\frac{1}{2}$$

12. Evaluate: $\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx$.

13. Evaluate: $\int x\sqrt{x^4-1}dx$.

Or

Evaluate: $\int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$.

14. Solve the following differential equation:

$$x^2\frac{dy}{dx}=2xy+y^2$$

Or

$$\cos^2 x\frac{dy}{dx}+y=\tan x$$

15. Find the derivative of $e^{\sqrt{x}}$ w.r.t. x from first principle.

16. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

17. A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant 40%. 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be of standard quality. Find the probability that it comes from the second plant.

Or

Six coins are tossed simultaneously. Find the probability of getting

(i) 3 heads

(ii) no heads

(iii) at least one head.

*18. Show that if $f: A \to B$ and $g: B \to C$ are one-one then $go f: A \to C$ is one-one.

19. Using the properties of determinants, evaluate the following:

$$\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & cb^2 & 0 \end{vmatrix}$$

20. Evaluate: $\int_{\pi/4}^{\pi/2} \cos 2x . \log \sin x \, dx$.

21. Using vectors, prove that the mid-point of hypotenuse of a right angled triangle is equidistant from its vertices.

*22. Write the following functions in the simplest form $\tan^{-1}\left(\sqrt{1+x^2}+x\right)$.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Find the vector equations of the line passing through the point A (2, –1, 1) and parallel to the line joining the points B (–1, 4, 1) and C (1, 2, 2). Also find the Cartesian equation of the line.

Or

Find the foot of the perpendicular from (0, 2, 7) on the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$.

24. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

25. A square piece of tin of side 18 cm is to be made into a box without a top by cutting a square place from each corner and folding up the flaps. What should be the side of the square to be cut off, so that the volume of the box be maximum? Also find the maximum volume of the box.

26. Obtain the inverse of the following matrix using elementary operations.

$$A + \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

27. Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$.

28. Find the equations of the plane passing through the point (1, 1, 1) and perpendicular to each of the following planes:

$$x + 2y + 3z = 7 \text{ and } 2x - 3y + 4z = 0$$

29. A company manufactures two articles A and B. There are two departments through which these articles are processed : (i) assembly and (ii) finishing departments. The maximum capacity of the first department is 60 hours a week and that of the other department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs 6 for each unit of A and Rs 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

Or

A factory owner wants to purchase two types of machines, A and B, for his factory. The machine A requires an area of 1000 m^2 and 12 skilled men for running it and its daily output is 50 units, whereas the machine B requires 1200 m^2 area and 8 skilled men, and its daily output is 40 units. If an area of 7600 m^2 and 72 skilled men be available to operate the machine, how many machines of each type should be bought to maximize the daily output?

ANSWERS

2. $\begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

4. $\frac{1}{\sqrt{1+x^2}}$

6. $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}=xy-x^2+2$

7. $\frac{7}{22}\text{cm/sec}$

8. $\hat{i}+2\hat{j}+\hat{k}$

9. $\frac{2\pi}{3}$

10. $-\frac{2}{7}\log|x+2|-\frac{3}{14}\log|3-2x|+c$

11. $f(x)$ is continuous at $x=\frac{1}{2}$

12. $\int\left(\frac{\pi}{4}-\frac{x}{2}\right)dx=\frac{\pi}{4}\cdot x-\frac{x^2}{4}+c$

13. $\frac{1}{4}\left[x^2\sqrt{x^4-1}-\log(x^2+\sqrt{x^4-1})\right]+c$

Or

$2\log|1+\sqrt{x}|+c$

14. Or

$y=\tan x-1+ce^{-\tan x}$

15. $\frac{e^{\sqrt{x}}}{2\sqrt{x}}$

16. $\frac{31}{153}$

17. $\frac{3}{7}$

Or

(i) $\frac{5}{16}$ (ii) $\frac{1}{64}$ (iii) $\frac{63}{64}$

19. $\Delta=a^2b^2c^2\cdot a\begin{vmatrix} b & -b \\ c & c \end{vmatrix}=2a^3b^3c^3$

20. $\frac{1}{4}\left(1-\frac{\pi}{2}+\log 2\right)$

22. $y=\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$

23. Vector Equation, $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$

Cartesian Equation, $\frac{x-2}{2}=\frac{y+1}{-2}=\frac{z-1}{1}(=\lambda)$

Or

$P\left(\frac{1}{2}-2,-\frac{3}{2}+1,1+3\right)=\left(-\frac{3}{2},-\frac{1}{2},4\right)$

24. $\frac{2}{5}$

25. Volume is maximum, when side of the square cut off is 3 cm.
Maximum value of the volume = $(18-2\times 3)^2\times 3 = 432\ cm^3$

26. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$

27. $\frac{16}{3}a^2$ sq.units

28. $17x + 2y - 7z = 12$

29. Maximum Value (= 120) obtain when no. of articles A is 12 and the no. Of articles B is 6.

Or

Maximum output (= 320) is obtain when no. of purchased machines A and B are respectively 4 and 3.