CBSE MATHS 2004 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*****1. Show that the relation R in the set {1, 2, 3}, given by R= {(1, 2), (2, 1)} is not reflexive.

*2. Using the property of determinants, prove that

$$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0 .$$

*3. Find the values of a, b, c and d from the following equations.

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}.$$

4. If $A = \begin{pmatrix} 3 & 5 \\ -4 & 2 \end{pmatrix}$, show that $A^2 - 5A - 141 = O$.

5. Evaluate: $\int \sin^4 x \, dx$.

6. Evaluate: $\int \frac{\sin^{-1} x}{x^2} dx$.

7. From the differential equation corresponding to $y^2 = a(b - x^2)$ where a and b are arbitrary constants.

*8. Find the principal values of $\tan^{-1}(-1)$.

9. If $\hat{a}$ and $\hat{b}$ and $\hat{b}$ are two unit vectors and $\theta\pi$ is angle between them, show that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$.

10. Evaluate: $\int \frac{1}{3 + 2\sin x + \cos x} dx$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. An urn contains 7 white, 5 black and 3red balls. Two balls are drawn at random. Find the probability that

(i) both the balls are red

(ii) one ball is red, the other is black

(iii) one ball is white

12. A fair die is tossed twice. If the number appearing on the top is less than 3, it is a success. Find the probability distribution of successes.

*13. Consider $f : N \to N$, $g : N \to N$ and $h : N \to R$ defined as $f(x) = 2x$, $g(y) = 3y + 4$ and $h(z) = \sin z \forall x$, y and z in N. Show that $h_0 (gof) = (hog)_0 f$.

14. Solve the differential equation:

$(1 + e^{2x})dy + e^x (1 + y^2) dx = 0$

Given that y = 1, when x = 0

Or

Solve the differential equation:

$$x\frac{dy}{dx} - y - 2x^3 = 0$$

*15. Show that the function:

$$f(x) = \begin{cases} \frac{\sin}{x} + \cos x, & x \neq 0 \\ 2 & x = 0 \end{cases}$$

is continuous at x = 0

Or

Find the value of K so that the function

$$f(x) = \begin{cases} Kx^2, & \text{if } x \leq 2 \\ 3 \quad , & \text{if } x > 2 \end{cases}$$

is continuous at x = 2.

16. Differentiate $\sqrt{osx}$ w.r.t. x from first principles.

17. Differentiate $\cot^{-1}\left(\frac{1-x}{1+x}\right)$ w.r.t. x.

18. Find the equations of the tangent and the normal to the curve $x = 1 - \cos\theta$, $y = \theta - \sin\theta$ at $\theta = \frac{\pi}{4}$.

19. Using properties of determinants, solve for x:

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$$

*20. Prove that

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3), 4 \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

21. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

Or

Find x such that the four points A (3, 2, 1), B (4, x, 5), C (4, 2, –2) and D (6, 5, –1) are coplanar.

22. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively.

Find P ($X \geq 1$).

Or

If the sum of the mean and variance of a binomial distribution for 5 trials be 1.8, find the distribution.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Evaluate: $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$.

*24. Find the angle between the pair of lines given by

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

25. Using matrix method solve the following system of linear equations:

$x + y + z = 3$

$2x - y + z = 2$

$x - 2y + 3z = 2$

26. Show that a right circular cylinder which is open at the top, and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

27. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

Or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$.

28. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find the point of intersection also.

29. An oil company requires 13,000, 20,000 and 15,000 barrels of high grades, medium grade and low grade oil respectively. Refinery A produces 100, 300, and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs 400 per day and B costs Rs 300 per day to operate, how many days should each be run to minimize the cost of requirement?

Or

A firm makes items A and B and the total number of items it can make in a day is 24. It takes one hour to make an item of A and only half an hour to make an item of B. The maximum time available per day is 16 hours. The profit on an item of A is Rs 300 and on one item of B is Rs 160. How many items of each type should be produced to maximize the profit? Solve the problem graphically.

ANSWERS

3. a = 1, b = 2, c = 3 and d = 4

5. $\frac{1}{8}\left(3-2\sin 2x+\frac{\sin 4x}{4}\right)+c$

6. $-\frac{\sin^{-1}x}{x}+\log\left|\frac{1-\sqrt{1-x^2}}{x}\right|+c$

7. $y\frac{dy}{dx}=x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]$

8. the principal value of $\tan^{-1}(-1)$ is $\frac{3\pi}{4}$

10. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+c$

11. (i) $\frac{1}{35}$ (ii) $\frac{1}{7}$ (iii) $\frac{8}{15}$

12. The probability distribution of X is

X	0	1	2
P(X)	$\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$	$\frac{1}{3}\times\frac{2}{3}+\frac{2}{3}\times\frac{1}{3}=\frac{4}{9}$	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

16. $\frac{dy}{dx}=\frac{-\sin\frac{2x}{2}}{[\sqrt{\cos x}+\sqrt{\cos x}]}\cdot 1=-\frac{\sin x}{2\sqrt{\cos x}}$

17. $-\frac{(1+x)^2}{2(1+x^2)}\cdot\frac{-2}{(1+x)^2}=\frac{1}{1+x^2}$

18. $-(\sqrt{2}+1)(\sqrt{2}x-\sqrt{2}+1)$

19. x = 0, 3a

21. Or

x = 5

22. $\frac{728}{729}$

Or

$\left(\frac{4}{5}+\frac{1}{5}\right)^5$

23. $\log\frac{4}{3}$

24. $\cos^{-1}\left(\frac{19}{21}\right)$

25. x = 1, y = 1, z = 1

27. $\frac{32}{3}\pi - \frac{4\sqrt{3}}{3} = \frac{4}{3}(8\pi - \sqrt{3})\text{sq.units}$

Or

$\frac{1}{4}ab(\pi - 2)$ sq. units

28. The point of intersection is (–1, –1, –1)

29. Refinery A should run for $\frac{170}{3}$ days and Refinery B should run for $\frac{110}{3}$ days.