CBSE MATHS 2005 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Show that the binary operation * defined by a * b = a – b, on z not commulative and associative.

*2. Find the values of a and b for which the following holds:

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

*3. If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ and B = [1 3 –6]. Find B' A'.

4. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, prove that

$A^3 - 4A^2 + A = O$.

5. Evaluate: $\int \frac{\sin 2x}{a^2 \sin^2 x + b^2 \cos^2 x} dx$.

6. Evaluate: $\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$.

7. Form the differential equations representing the family of curves $y^2 - 2ay + x^2 = a^2$, where a is an arbitrary constant.

*8. Write the following functions in the simplest form:

$$\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}, x < \pi$$

9. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/sec. Find the rate at which the volume of the bubble is increasing at the instant its radius is 6 cm.

10. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ be coplanar, show that $c^2 = ab$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. Let $f : x \to y$ and $g : y \to z$ be two invertible functions.
Then $go\,f$ is also invertible with $(gof)^{-1} = f^{-1}\,og^{-1}$

12. In a single throw of three dice, determine the probability of getting (a) a total of 5, (b) a total of at most 5.

13. A class consists of 10 boys and 8 girls. Three students are selected at random. Find the probability that the selected group has
(i) all boys,
(ii) all girls
(iii) 2 boys and 1 girl

14. Solve the following differential equation:

$$(1+x^2)\frac{dx}{dy} - 2xy = (x^2+2)(x^2+1)$$

Or

Solve the following differential equation:

$$x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$$

*15. Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$$

is continuous at x = 3.

Or

*Discuss the continuity of the function

$$f(x) = \begin{cases} \dfrac{x}{|x|+2x^2}, & x \ne 0 \\ 2, & x = 0 \end{cases}$$

at x = 0.

16. Differentiate $\sin\sqrt{x}$ w.r.t. x from first principles.

17. If $x = a\left(\frac{1+t^2}{1-t^2}\right)$ and $y = \frac{2t}{1-t^2}$ find $\frac{dy}{dx}$.

18. Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$,

Where a, b, c are in A.P.

19. Evaluate: $\int \frac{2x-1}{(x-1)(x+2)(x-3)}dx$.

20. Find the co-ordinates of the foot of the perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1).

Or

Find the equation of the plane passing through the line of intersection of the planes 2x + y – z = 3, 5x – 3y + 4z + 9 = 0 and parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{5}$.

21. The probability that student entering a university will graduate is 0.4. Find the probability that out of 3 students of the university:

(i) none will graduate

(ii) only one will graduate

(iii) all will graduate

*22. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Express the vector $\vec{a}=5\hat{i}-2\hat{j}+5\hat{k}$ and sum of two vectors such that one is parallel to the vector $\vec{b}=3\hat{i}+\hat{k}$ and the other is perpendicular to $\vec{b}$.

24. A company has two plants to manufacture motor cycles. 70% motor cycles are manufactured at the first plant, while 30% are manufactured at the second plant. At the first plant, 80% motor cycles are rated of the standard quality while at the second plant, 90% are rated of standard quality. A motor cycle, randomly picked up, is found to be of standard quality. Find the probability that it has come out from the second plant.

25. Using matrices, solve the following system of linear equations:

$x + y + z = 4$

$2x - y + z = -1$

$2x + y - 3z = -9$

Or

If $A=\begin{bmatrix}3 & 1\\ 7 & 5\end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

26. A wire the length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum.

27. Find the area bounded by the curve $x^2 = 4y$ and the straight line $4y - 2 = x$.

Or

Evaluate the following as limit of sums:

$$\int_0^2 (x^2+x)dx$$

28. Find the Cartesian as well as the vector equation of the planes passing through the intersection of the planes passing through the intersection of the planes $\vec{r}\cdot(\hat{2i}+6\hat{j})$ + 12 = 0 and $\vec{r}\cdot(3\hat{i}-\hat{j}+4\hat{k})=0$, which are at unit distance from the origin.

29. Solve the following linear programming problem graphically:

    Maximize z = 60x + 15y

    Subject to constraints

Or

Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

ANSWERS

2. $a = 1, b = -3$

3. $B'A' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$

5. $\frac{1}{a^2 - b^2} \log|a^2 \sin^2 x + b^2 \cos^2 x| + c$

6. $\frac{\log x\sqrt{16(\log x)^2}}{2} + 8\log\left|\log x + \sqrt{16 + (\log x)^2}\right| + c$

7. $x^2\left[1 - \left(\frac{dy}{dx}\right)^2\right] + 2y\left(2x + y\frac{dy}{dx}\right)\frac{dy}{dx} = 0$

8. $\frac{x}{2}$

9. 6 cm^3/sec

12. (i) $\frac{1}{36}$, (ii) $\frac{5}{108}$

13. (i) $\frac{5}{34}$, (ii) $\frac{7}{102}$, (iii) $\frac{15}{34}$

15. $a = \frac{3b+2}{3} = b + \frac{2}{3}$

Or

$f(x)$ is not continuous at $x = 0$

16. $\frac{\cos\sqrt{x}}{2\sqrt{x}}$

17. $\frac{1+t^2}{2at}$

19. $-\frac{1}{6}\log|x-1| - \frac{1}{3}\log|x+2| + \frac{1}{2}\log|x-3| + c$

Or

$\frac{\pi}{3}$

20. $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

Or

$7x + 9y - 10z - 27 = 0$

21. (i) 0.216, (ii) 0.432, (iii) 0.064

22. $\frac{1}{5}$

23. $-\hat{i}-2\hat{j}+3\hat{k}$

24. $\frac{27}{83}$

25. x = –1, y = 2, z = 3

Or

$$A^{-1}=\frac{1}{8}\begin{bmatrix}5 & -1\\ -7 & 3\end{bmatrix}$$

26. $\frac{324}{9+4\sqrt{3}}\text{cm}$

27. $\frac{9}{8}\text{sq.units}$

Or

$\frac{14}{3}$

28. x – 2y + 2z – 3 = 0,

Vector equations are $\vec{r}\cdot(2\hat{i}+\hat{j}+2\hat{k})+3=0$ and $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})-3=0$

29. Maximum value of z is at (30, 0)

Or

x = no. of days tailor A works, y = no. of days tailor B works, then

$x \geq 0, y \geq 0$