CBSE MATHS 2006 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. If $f : R \to R$ be given by $f(x) = (3 - x^3)^{1/3}$, then find the value of $fof(x)$.

*2. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, verify that $A^2 = 1$.

*3. Find the value of if $\begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$.

4. Solve the following differential equation:

$$\frac{dy}{dx} - \frac{y}{x} = 2x^2$$

5. Evaluate: $\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$.

6. Evaluate: $\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$.

*7. Simplify: $\tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$

*8. Verify Rolle's theorem for the following function : $f(x) = (x - 1)(x - 2)^2$, [1, 2].

9. Find the value of p so that the vectors $\hat{i} + 2\hat{j} - 3\hat{k}$, $p\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar.

10. If the mean and variance of the binomial distribution are respectively 9 and 6, find the distribution.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. Express the following matrix as the sum of a symmetric and a skew symmetric matrix.

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$$

*12. Find the inverse of the function $f(x) = 4x - 7$, $x \in R$.

13. From the differential equation of the family of curves $y = a \sin (x + b)$, where a and b are arbitrary constants.

Or

Solve the following differential equation:

$$2xy\, dx + (x^2 + 2y^2)\, dy = 0$$

14. Two dice are rolled once. Find the probability that:

(i) the number on two dice are different

(ii) the total of numbers on the two dice is at least 4.

15. Differentiate $\sin (2x + 3)$ w.r.t. x from first principle.

16. If $y = \sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$, find $\dfrac{dy}{dx}$ y.

Or

Find the value of m such that the function $f(x) = \begin{cases} m(x^2 - 2x), & x < 0 \\ \cos x \quad , & x \geq 0 \end{cases}$ is continuous at x = 0.

17. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are perpendicular to each other.

*18. Solve the following equation: $2 \tan^{-1} (\cos x) = \tan^{-1} (2 \operatorname{cosec} x)$

19. Using properties of determinants, prove the following:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

20. Evaluate: $\int_0^{\pi/4} \sin 2x \sin 3x \; dx$.

Or

Evaluate: $\int_0^{\pi/4} \log(1 + \tan x) dx$.

21. Evaluate: $\int \dfrac{3x+1}{2x^3 - 2x + 3} dx$

22. Find the equation of the plane passing through the points (1, 2, 3) and (0, –1, 0) and parallel to the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z}{-3}$

Or

*Find the values of P so that the lines

$$\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2} \quad \text{and} \quad \frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$$

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. A pair of dice is tossed twice. If the random variable X is defined as the number of doublets, find the probability distribution of X.

*24. Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=0$. Evaluate the quantity.

$\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}, \text{if } |\vec{a}|=1, |\vec{b}|=4 \text{ and } |\vec{c}|=2$

25. Using matrices, solve the following system of equations:

$x + y + z = 3$; $x – 2y + 3z – 2$ and $2x – y + z = 2$.

26. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, –8).

Or

Prove that the height of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

27. Find the area of the region bounded by $y^2 = 4x$, $x = 1$, $x = 4$ and x-axis in the first quadrant.

Or

Evaluate $\int_0^2 (x^2+x+1)dx$ as limit of a sum.

28. The vector equations of two lines are:

$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$.

Find the shortest distance between the above lines.

29. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically.

ANSWERS

1. $(f o f) = x$
3. $x = 12$
4. $y = x^3 + cx$
5. $\dfrac{\left(x-\frac{3}{2}\right)\sqrt{x^2-3x+2}}{2}-\dfrac{1}{8}\log\left|\left(x-\dfrac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+c$
6. $x\cos 2\alpha - \sin 2\alpha \log|\sin(x+\alpha)| + c$
7. $\tan^{-1}\dfrac{a}{b} - x$
8. $c = \dfrac{4}{3} \in]\,1, 2\,[$
9. $p = 2$
10. $(q+p)^n = \left(\dfrac{2}{3}+\dfrac{1}{3}\right)^{27}$
11. $= \dfrac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$
12. $f^{-1}(x) = \dfrac{x+7}{4}$
13. $\dfrac{d^2y}{dx^2} + y = 0$

    Or

    $3x^2y + 2y^3 = c$
14. (i) $\dfrac{5}{6}$, (ii) $\dfrac{11}{12}$
15. $2\cos(2x + 3)$
16. $\dfrac{dy}{dx} = \sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}\cdot\left[\dfrac{1}{2}\cdot\dfrac{1}{x-3}+\dfrac{x}{x^2+4}-\dfrac{3x+2}{3x^2+4x+5}\right]$

    Or

    $f(x)$ cannot be continuous at $x = 0$ for any value of m.
18. $x = 0$ or $x = \dfrac{\pi}{4}$
20. $\dfrac{3}{5\sqrt{2}}$

Or

$\frac{\pi}{8}\log 2$

21. $\frac{3}{4}\log|2x^2-2x+3|+\frac{\sqrt{5}}{2}\tan^{-1}\left(\frac{2x-1}{\sqrt{5}}\right)+c$

22. 6x – 3y + z = 3

Or

$P=\frac{70}{11}$

23. The required probability distribution is

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

24. $\mu=-\frac{21}{2}$

25. x = 1, y = 1, z = 1

26. (4, –4) is the point on $y^2 = 4x$, nearest to the given point A (2, –8).

Or

$\frac{4\pi R^3}{3\sqrt{3}}$

27. $\frac{28}{3}$ sq. units

Or

$\frac{20}{3}$

28. $\frac{3}{\sqrt{19}}$ units

29. The profit is maximum at R (8, 12)
When 8 fans and 12 sewing machines are purchased and sold and the max profit is Rs 392.