CBSE MATHS 2007 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

*1. Show that the binary operation * defined by a * b = ab + 1 on Q is not associative.

*2. If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and B = [1 0 4] find (AB).

*3. Find the area of the triangle whose vertices are (2, 7), (1, 1) and (10, 8).

4. Evaluate: $\int \frac{1+x^2}{1+x^4} dx$

5. Solve the following differential equation:

$$X \cos y \, dy = (xe^x \log x + e^x)\, dx$$

6. Evaluate: $\int \cos 4x \cos 3x \, dx$.

7. Find the value of k if the function $f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases}$ is continuous at x = 1.

*8. Write into the simplest form:

$$\tan^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$$

9. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

10. Find the value of λ which makes the vectors $\vec{a}, \vec{b}, \vec{c}$ coplanar, where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show that $A^2 - 6A + 17I = 0$. Hence find A^{-1}.

12. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is neither an ace nor a king.

13. Form the differential equation of the family of curves y = A cos 2x + B sin 2x, where A and B are constants.

Or

Solve the following differential equation:

$$\frac{dy}{dx} + 2y = 6e^x$$

*14. Discuss the continuity of the function

$$f(x) = \begin{cases} x+1 & \text{if } x \geq 1 \\ x^2+1 & \text{if } x < 1 \end{cases}$$

at x = 1

15. Differentiate sin $(x^2 + 1)$ with respect to x from first principle.

16. If f be a greatest integer function and g be an absolute value functions. Find the value of

$$(fog)\left(\frac{-3}{2}\right) + (gof)\left(\frac{4}{3}\right)$$

17. If y = sin (log x), prove that

$$x^2 \cdot \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Or

Verify Rolle's theorem for the function:

$$f(x) = x^2 - 5x + 4 \text{ on} [1, 4]$$

*18. Write into the simplest form:

$$\cot^{-1}\left(\sqrt{1+x^2} - x\right)$$

19. Using properties of determinants, prove the following:

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

20. Using properties of definite integrals prove the following:

$$\int_0^{\pi} \frac{x\tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

Or

Evaluate: $\int \frac{\sin x}{(1-\sin x)(2-\cos x)} dx$

*21. Find the equation of a line, which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

22. Find mean μ, variance σ^2 for the following probability distribution:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Or

Find the binomial distribution for which the mean is 4 and variance 3.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting (a) 2 red balls (b) 2 blue balls (c) one red and one blue ball.

24. Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

25. Using matrices, solve the following system of equations:

$$x + 2y - 3z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

26. Using integration, find the area of the region enclosed between the circles:

$$x^2 + y^2 = 1 \text{ and } (x-1)^2 + y^2 = 1$$

Or

$\int_0^2 (x^2 + 2x + 1)dx$ as limit of sums.

27. Find the point on the curve $x^2 = 8y$ which is nearest to the point (2, 4).

Or

Show that the right circular cone of least curved surface and given value has an altitude equal to $\sqrt{2}$ times the radius of the base.

28. A line makes angle α, β, γ and δ with the diagonals of a cube. Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$

29. If a young man rides his motorcycle at 2.5 km/hour, he had to spend Rs 2 per km on petrol. If he rides at a faster speed of 40 km/hour, the petrol cost increases at Rs 5 per km. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.

ANSWERS

2. $(AB)' = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$

3. 23.5 sq. units

4. $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$

5. $e^x \log x + c$

6. $\frac{1}{14}\sin 7x + \frac{1}{2}\sin x + c$

7. k = 4

8. $\frac{x}{2}$

9. 2

10. $\lambda = 4$

11. $\frac{1}{17}\begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$

12. $\frac{11}{13}$

13. $\frac{d^2y}{dx^2} + 4y = 0$

Or

$2e^x + ce^{-2x}$

14. $f(x)$ is continuous at x = 1

15. $2x\cos(x^2 + 1)$

16. 2

17. Or

$\frac{5}{2} \in (1, 4)$

18. $\frac{\pi}{4} + \frac{1}{2}\tan^{-1} x$

20. Or

$\log\left|\frac{1 - \cos x}{2 - \cos x}\right| + c$

21. $\frac{x-1}{-2} = \frac{y-2}{14} = \frac{z-3}{3}$

22. 0.75

Or

Binomial Distribution = $\left(\frac{3}{4}+\frac{1}{4}\right)^{16}$

23. (a) $\frac{49}{121}$, (b) $\frac{16}{121}$, (c) $\frac{56}{121}$

24. $47x + 15y - 50z + 173 = 0$

25. $x = 1, y = -5, z = -5$

26. $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ sq. unit

Or

$\frac{26}{3}$

27. (4, 2)

29. The coordinates of feasible region is A (0, 20), $P\left(\frac{50}{3},\frac{40}{3}\right)$, D (25, 0)

$\frac{50}{3}$ km at the speed of 25 km/hour and $\frac{40}{3}$ km at the speed of 40 km/hour.