CBSE MATHS 2008 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

1. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in R$, find (fog) (7).

2. Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

3. Find the value of x and y if:

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

4. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

5. Find the co-factor of a_{12} in the following:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

6. Evaluate: $\int \frac{x^2}{1+x^3} dx$.

7. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$.

8. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

9. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$.

10. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. (i) Is the binary operation *, defined on set N, given by $a * b = \frac{a+b}{2}$ for all a, b $\in$ Q, commutative?

(ii) is the above binary operation * associative?

12. Prove that following:

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$$

13. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

Or

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = 0$.

14. For what value of k is the following function continuous at x = 2?

15. Differentiate the following with respect to x:

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

16. Find the equation of tangent to the curve x = sin 3t, y = cos 2t, at t = $\pi/4$.

17. Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$.

18. Solve the following differential equation:

$(x^2 - y^2)dx + 2xy\, dy = 0$ given that y = 1 when x = 1.

Or

Solve the following differential equation:

$\frac{dy}{dx}=\frac{x(2y-x)}{x(2y+x)}$, if y = 1 when x = 1.

19. Solve the following differential equation:

$$\cos^2 x\frac{dy}{dx}+y=\tan x$$

20. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$.

Or

If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|$ = 7, show that the angle between $\vec{a}$ and $\vec{b}$ is 60^o.

21. Find the shortest distance between the following lines:

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1} \text{ and } \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$

Or

Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).

22. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Using properties of determinants, prove the following:

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$

24. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

Or

Show that the height of the cylinder of maximum value that can be inscribed in a cone of height h is $\frac{1}{3}$ h.

25. Using integration find the area of the region bounded by the parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$.

26. Evaluate: $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx$.

27. Find the equation of the plane passing through the point (–1, –1, 2) and perpendicular to each of the following planes:
$2x + 3y – 3z = 2$ and $5x – 4y + z = 6$

Or

Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$

28. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines are as follows:

Machine	Area Occupied	Labour Force	Daily Output (in units)
A	1000 m^2	12 men	60
B	1200 m^2	8 men	40

He has maximum area of 9000 m^2 available, and 72 skilled labours who can operate both the machines. How many machines of each type should he buy to maximize the daily output?

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver.

ANSWERS

1. 7
2. 1
3. $x = 3, y = 3$
4. $a^2 + b^2 + c^2 + d^2$
5. 46
6. $\frac{1}{3}\log|1 + x^3| + c$
7. $\frac{\pi}{4}$
8. $\frac{3}{7}\hat{c} - \frac{2}{7}\hat{i} + \frac{6}{7}\hat{k}$
9. $\cos^{-1}\left(\frac{-1}{3}\right)$
10. $\lambda = \frac{5}{2}$