CBSE MATHS 2009 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\cdot\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write a unit vector in the direction of $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$.

3. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

4. If matrix A = (1 2 3), write AA', where A' is the transpose of matrix A.

5. Write the value of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

6. Using principal value, evaluate the following: $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$

7. Evaluate: $\int \frac{\sec^2 x}{3+\tan x}dx$.

8. If $\int_0^1 (3x^2 + 2x + k)dx = 0$, find the value of k.

9. If the binary operation * on the set integers Z, is defined by a * b = a + $3b^2$, then find the value 2 * 4.

10. If A is an invertible matrix of order 3 and |A| = 5, then find |adj. A|.

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$, where $\vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$.

12. Prove that: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$.

Or

Solve for: $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4}$.

13. Find the value of λ so that the lines $\frac{1-x}{3}=\frac{7y-14}{2\lambda}=\frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other.

14. Solve the following differential equation: $\frac{dy}{dx}+y=\cos x-\sin x$.

15. Find the particular solution, satisfying the given condition, for the following differential equation:

$$\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0;\ y=0 \text{ when } x=1.$$

16. By using properties of determinants prove the following:

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}=(5x+4)(4-x)^2.$$

17. A die thrown again and until three sixes are obtained. Find the probability of obtaining the third in the sixth throw of the die.

18. Differentiate the following function w.r.t. x: $x^{\sin x}+(\sin x)^{\cos x}$.

19. Evaluate: $\int\frac{e^x}{\sqrt{5-4e^x-e^{2x}}}dx$

Or

$\int\frac{(x-4)e^x}{(x-2)^3}dx$

20. Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation.

21. Find $\frac{dy}{dx}$ if $(x^2+y^2)^2=xy$.

Or

If $y=3\cos(\log x)+4\sin(\log x)$, then show that $x^2\cdot\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

22. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line 4x – 2y + 5 = 0.

Or

Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing.

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. Find the volume of the largest cylinder that can be inscribed in a sphere of radius r.

Or

A tank with rectangular base and rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs. 70 per sq. metre for the base and Rs. 45 per sq. metre for sides, what is the cost of least expansive tank?

24. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1 costs Rs. 4 per unit and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of Vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

25. Three bags contain balls as shown in the table below:

Bag	Number of White balls	Number of Black balls	Number of Red balls
I	1	2	3
II	2	1	1
III	4	3	2

A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag

26. Using matrices, solve the following system of equations:

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

27. Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$.

Or

Evaluate: $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x) dx$.

28. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

29. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

ANSWERS

1. $\frac{8}{7}$

2. $= \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$

3. $p = \frac{2}{3}$

4. AA = [14]

5. 0 [The elements R_1 and R_3 are same]

6. $\sin^{-1}\sin\frac{2\pi}{5} = \frac{2\pi}{5}$

7. $= \log|3 + \tan x| + c$

8. k = –2

9. 50

10. 25

13. 7

14. $y = \cos x + ce^{-x}$

15. 1 + log x

16. No answer

17. $\frac{625}{23328}$

18. $x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right] + (\sin x)^{\cos x}[\cos x \cot x - \sin x \log\sin x]$

19. No answer

OR

$\frac{e^x}{(x-2)} + c$

21. $\frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x}$

22. 48x – 24y – 23 = 0

OR

$f(x)$ is increasing on $(-\infty, -1] \cup [1, \infty)$

$f(x)$ is decreasing on $[-1, 0) \cup (0, 1]$

23. OR

Rs 1000

24. The minimum cost is Rs. 104 when x = 24units and y = $\frac{4}{3}$ units

27. $I = \frac{\pi}{2}$

OR

$I = -\frac{\pi}{2}\log 2$

28. $\frac{7}{2}$ sq. units

29. $7x - 8y + 3z + 25 = 0$