CBSE MATHS 2010 YEAR PAPER

Important Instructions:

(i) The question papers consists of three sections A, B and C.

(ii) All questions are compulsory.

(iii) Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.

(iv) Use of calculators is not permitted. However, you may ask for logarithmic and statistical tables, if required.

(v) Questions with * are now OUT Of COURSE.

SECTION – A

Question numbers 1 to 10 carry 1 mark each

1. What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?

2. What is the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$?

3. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what of α is A an identity matrix?

4. What is the value of the determinant $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$?

5. Evaluate: $\int \frac{\log x}{x} dx$

6. What is the degree of the following differential equation?

$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$$

7. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$.

8. Write the vector equation of the following line:

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$$

9. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of k.

10. What is the cosine of the angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with y-axis?

SECTION – B

Question numbers 11 to 22 carry 4 marks each

11. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

12. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of the line segment RQ.

13. Find the Cartesian equation of the plane passing through the points A (0, 0, 0) and B (3, –1, 2) and parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$.

14. Using elementary row operations, find the inverse of the following matrix:

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

15. Let Z be the set of all integers and R be the relation on Z defined as R = {(a, b); a, b ∈ Z, and (a – b) is divisible by 5.} Prove that R is an equivalence relation.

16. Prove that following:

$$\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), x\in(0,1)$$

Or

Prove the following:

$$\cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

17. Show that the function defined as follows, is continuous x = 2, but not differentiable thereat:

$$f(x)=\begin{cases} 3x-2 & , \quad 0<x\le 1 \\ 2x^2-x & , \quad 1<x\le 2 \\ 5x-4 & , \quad x>2 \end{cases}$$

Or

Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$

18. Evaluate: $\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

Or

Evaluate: $\int\frac{1-x^2}{x(1-2x)}dx$

19. Evaluate: $\int_{\pi/6}^{\pi/3}\frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$.

20. Find the points on the curve $y = x^2$ at which the slope of the tangent is equal to the y-coordinate of the point.

21. Find the general solution of the differential equation

$$x \log x \cdot \frac{dy}{dx} + y = \frac{2}{x} \cdot \log x$$

Or

Find the particular solution of the differential equation satisfying the given conditions:

$\frac{dy}{dx} y \tan x$, given that y = 1, when x = 0.

22. Find the particular solution of the differential equation satisfying the given conditions:

$x^2 dy + (xy + y^2) dx = 0$

SECTION – C

Question numbers 23 to 29 carry 6 marks each

23. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as L.P.P. and solve it graphically.

24. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to both clubs. Find the probability of the lost card being of clubs.

Or

From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

25. The points A(4, 5, 10), B (2, 3, 4) and C(1, 2, –1) are three vertices of a parallelogram ABCD. Find the vector equations of the sides AB and BC and also find the coordinates of point D.

26. Using integration, find the area of the region bounded by the curve $x^2 = 4y$ and the line x = 4y – 2.

27. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

28. Find the values of x for which $f(x) = [x(x - 2)]^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

29. Using properties of determinants, show the following:

$$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

ANSWERS

1. Range = {–1, 1}
2. $-\frac{\pi}{3}$
3. $\alpha = 0^o$
4. 8
5. $\frac{(\log x)^2}{2} + c$
6. Degree = 1
7. $5\hat{i} - 10\hat{j} + 10\hat{k}$
8. $\left(5\hat{i} - 4\hat{j} + 6\hat{k}\right) + \lambda\left(3\hat{i} + 7\hat{j} - 2\hat{k}\right)$
9. $\begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$
10. The cosine of angle which the given vector makes with y-axis is $\frac{1}{2}$
12. $2\vec{a} + \vec{b}$, hence point P is the mid point of the line segment RQ.
13. x – 19y – 11z = 0
14. $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$
16. $\tan^{-1}\sqrt{x}$

    Or

    $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$
17. $f(x)$ is continuous at x = 2, not differentiable at x = 2
18. $I = e^x \cot 2x + C$

    OR

    $= \frac{1}{2}x + \log|x| - \frac{3}{4}\log|1 - 2x| + C$
19. $2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$
20. Required points (0, 0), (3, 27)
21. $y\log x = \frac{-2\log x}{x} - \frac{2}{x} + C$

    $xy \log x = -2\log x - 2 + Cx$

    OR

    $y = \sec x$

22. $y + 2x = 3x^2$

24. $\frac{11}{50}$

OR

x	0	1	2
P(x)	$\frac{14}{30}$	$\frac{14}{30}$	$\frac{2}{30}$

25. $x = 3, y = 4, z = 5$

Hence coordinates of D are (3, 4, 5)

26. $\frac{9}{8}\text{sq, units}$

28. required points are (0, 0) (1, 1) and (2, 0)